GRADE : 12	DATE : 26 / 5 / 2014
SUBJECT : Mathematics	**SOLUTIONS**
TITLE : June Paper 1	
EXAMINER : Mr A. Slaughter	
TOTAL MARKS : 150	TIME : 3 hour(s)

1.1.	1. $5x^2 - 4x - 11 = 0$	
	$(\quad)(\quad) = 0$??	
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
	✓ $= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-11)}}{2(5)}$	
	$= \frac{4 \pm \sqrt{236}}{10}$	
	$= 1{,}94$ ✓ or $-1{,}14$ ✓	**3**
1.1.	2. $0 > -2(2x-5)(x+3)$	
	$\div -2$: $0 < (2x-5)(x+3)$	
	$+ \quad 0 \quad - \quad 0 \quad +$; -3, $5/2$	
	$x < -3$ ✓ or ✓ $\frac{5}{2} < x$ ✓	**3**
1.1.	3. Ignore signs $\frac{3}{2} > \frac{3}{4}$	
	$\therefore 6x^{-3/2} - 7x^{-3/4} - 3 = 0$	
	$k = x^{-\frac{3}{4}}$ $\therefore (k)^2 = (x^{-\frac{3}{4}})^2$	
	$= x^{-3/2}$	
	$6k^2 - 7k - 3 = 0$	
	$(2k-3)(3k+1) = 0$ ✓	
	$k = 3/2$ or $-\frac{1}{3}$ ✓	
	$x^{-\frac{3}{4}} = \frac{3}{2}$ or $x^{-\frac{3}{4}} = -\frac{1}{3}$	
	$\left(x^{-\frac{3}{4}}\right)^{-\frac{4}{3}} = \left(\frac{3}{2}\right)^{-\frac{4}{3}}$ ✓ ; no soln ✓	
	$x = 0{,}58$ ✓	**5**
1.2.	$3x^2 + 11xy - 4y^2 = 0$	
	$(3x - y)(x + 4y) = 0$ ✓	
	$\therefore 3x - y = 0$ or $x + 4y = 0$	
	$3x = y$; $x = -4y$	
	$\frac{x}{y} = \frac{1}{3}$ ✓ ; $\frac{x}{y} = -4$ ✓	**3**
1.3.	$4y - x = -6$	
	$4y + 6 = x$ ✓	
	$3y^2 - (4y+6)^2 - 2(4y+6)y = 3$ ✓	
	$3y^2 - (16y^2 + 48y + 36) - 2(4y^2 + 6y) = 3$	
	$3y^2 - 16y^2 - 48y - 36 - 8y^2 - 12y - 3 = 0$	
	$-21y^2 - 60y - 39 = 0$	
	$\div -3$: $7y^2 + 20y + 13 = 0$ ✓	

$(7y+13)(y+1)=0$

$\therefore\ y = -\frac{13}{7}$ or -1

$\therefore\ x = 4(-\frac{13}{7})+6$ or $4(-1)+6$

$= -\frac{10}{7}$ $\quad = 2$

So,

$x = 2$ and $y = -1$

or

$x = -\frac{10}{7}$ and $y = -\frac{13}{7}$

7

1.4. 1. $9 = 3^2$

$\therefore\ \frac{(3^2)^{1006}}{3^{2015}+3^{2013}}$

$= \frac{3^{2012}}{3^{2012}.3^3 + 3^{2012}.3^1}$

$= \frac{3^{2012}}{3^{2012}(3^3+3)}$

$= \frac{1}{27+3}$

$= \frac{1}{30}$

3

$\cancel{\frac{3^{2012}}{3^{2013}(3^2+1)}}$ $\quad 3^1.3^{2012}$

$= \frac{1}{3(9+1)}$

$= \frac{1}{30}$

1.4. 2. $(\sqrt[3]{54} - \sqrt[3]{250})^3$

$\sqrt[3]{54} = \sqrt[3]{27.2}$

$= \sqrt[3]{27}.\sqrt[3]{2}$

$= 3.\sqrt[3]{2}$ 0 if 27.2 not shown

$\sqrt[3]{250} = \sqrt[3]{125.2}$

$= \sqrt[3]{125}.\sqrt[3]{2}$

$= 5.\sqrt[3]{2}$ 0 if 125.2 not shown

$\therefore\ (3\sqrt[3]{2} - 5\sqrt[3]{2})^3$

$= (-2\sqrt[3]{2})^3$

$= -8.2$

$= -16$

4

2.1. 1. $x^2+bx-2+k(x^2+3x+2)=0$

$x^2+bx-2+kx^2+3kx+2k=0$

$x^2+kx^2+bx+3kx-2+2k=0$

$x^2(1+k)+x(b+3k)+(-2+2k)=0$

Δ

$= b^2-4ac$

$= (b+3k)^2-4(1+k)(-2+2k)$

$= b^2+6bk+9k^2-4(-2+2k-2k+2k^2)$

$= b^2+6bk+9k^2-4(2k^2-2)$

$= b^2+6bk+9k^2-8k^2+8$

$= b^2+6bk+k^2+8$

3

2.1.	2. $\Delta = k^2 + 6bk + b^2 + 8$	
	$b = 0$	
	$\therefore \Delta = k^2 + 6(0)k + (0)^2 + 8$	
	$= k^2 + 8$	
	Now, $k^2 \geq 0$	
	$k^2 + 8 \geq 8$	
	$k^2 + 8 > 0$	
	$\therefore \Delta > 0$ ✓	
	$\therefore$ roots will be	
	• real ✓	
	• unequal ✓	**3**
	(In some instances	
	eg $k = 1$, then	
	$\Delta = 1^2 + 8$	
	$= 9$	
	so the roots will	
	be rational)	**3**
2.2.	$f: y = -2x^2 - x + 3$	
	$g: y = 3x + c$	
	$\therefore 3x + c = -2x^2 - x + 3$ ✓	
	$2x^2 + 4x + c - 3 = 0$ ✓	
	$\Delta = (4)^2 - 4(2)(c-3)$ ✓	
	$= 16 - 4(2c - 6)$	

	$= 16 - 8c + 24$	
	$= 40 - 8c$	
	tangential : $\Delta = 0$ ✓	
	$40 - 8c = 0$	
	$c = 5$ ✓	**5**
3.1.	$3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} \ldots$	
3.1.1.	$a = 3 \quad r = \frac{-\frac{3}{2}}{3}$	
	✓ $r = -\frac{1}{2}$	
	$T_n = ar^{n-1}$	
	$T_{20} = 3.(-\frac{1}{2})^{19}$ ✓ f+s	
	$= -\frac{3}{524\,288}$ ✓	**3**
3.1.2.	$S_n = \frac{a(r^n - 1)}{r - 1}$	
	fs	
	$S_{15} = \frac{3((-\frac{1}{2})^{15} - 1)}{-\frac{1}{2} - 1}$ ✓	
	$= \frac{32\,769}{16\,384}$ ✓	**2**

3.2.	$-\frac{1}{2}; 10; \frac{1}{2}; 10; \frac{3}{2}; 10; \quad 10 \quad 10$		3.3.	18 20 22 24 26 28 30 4998 5000	
	1 2 3 4 5 6 98 99 100				
				$18; 24; 30; \ldots; 4998$ ✓ ✓	
	$-\frac{1}{2} \quad \frac{1}{2} \quad \frac{3}{2}$				
	1 2 3 50			$a = 18 \quad d = 6$ ✓ad	
	10 10 10 10 10			$T_n = a + (n-1)d$	
	1 2 3 49 50			$4998 = 18 + (n-1)(6)$ ✓fs	
				$4980 = (n-1)(6)$	
				$830 = n-1$	
3.2.	1. $T_{100} = 10$ ✓	1		$831 = n$ ✓	5
3.2.	2. $a = -\frac{1}{2} \quad d = 1$ ✓ad		4.1.	$3,\dot{7}\dot{5}$	
	$S_n = \frac{n(2a + (n-1)d)}{2}$			$= 3,757575$	
	$S_{50} = \frac{50(2(-\frac{1}{2}) + 49(1))}{2}$ ✓ ✓fs			$= 3$	
	$= 1200$			$+ 0,75$	
				$+ 0,0075$	
				$+ 0,000075$	
	$a = 10 \quad d = 0$			$a = 0,75 \quad r = \frac{0,0075}{0,75}$	
	$\therefore S_{49} = 49.10$ ✓			a,r ✓ $= \frac{1}{100}$	
	$= 490$				
				$S_\infty = \frac{a}{1-r}$	
	$\therefore S_{99} = 1200 + 490$			$= \frac{0,75}{1 - \frac{1}{100}}$ ✓	
	$= 1690$ ✓	5		$= \frac{25}{33}$	
				design	
				$\therefore 3,\dot{7}\dot{5} = 3 + \frac{25}{33}$ ✓	
				$= \frac{124}{33}$ ✓	4

4.2. $\sum_{k=3}^{\infty} (1-2x)^k$

4.2.1. $(1-2x)^3 + (1-2x)^4 + (1-2x)^5$ **1**

4.2.2. $r = \frac{(1-2x)^4}{(1-2x)^3}$

$= 1-2x$

For convergence

$-1 < r < 1$ and $r \neq 0$

$-1 < 1-2x < 1$ $1-2x \neq 0$

$-2 < -2x < 0$ $x \neq \frac{1}{2}$

$1 > x > 0$ **4**

5. $-10; -6; 4; 20; \ldots$

4 10 16

6 6

$d_2 = 2a$ $d_1 = 3a+b$ $T_1 = a+b+c$

$6 = 2a$ $4 = 3(3)+b$ $-10 = 3+(-5)+c$

$3 = a$ $-5 = b$ $-8 = c$

$\therefore T_n = 3n^2 - 5n - 8$ **4**

6.1. $g: y = -3^{x+1} + 9$

$= -1 . 3^{x+1} + 9$

yint: $y = -1 . 3^{0+1} + 9$

$= 6$

xint: $0 = -1 . 3^{x+1} + 9$

$-9 = -1 . 3^{x+1}$

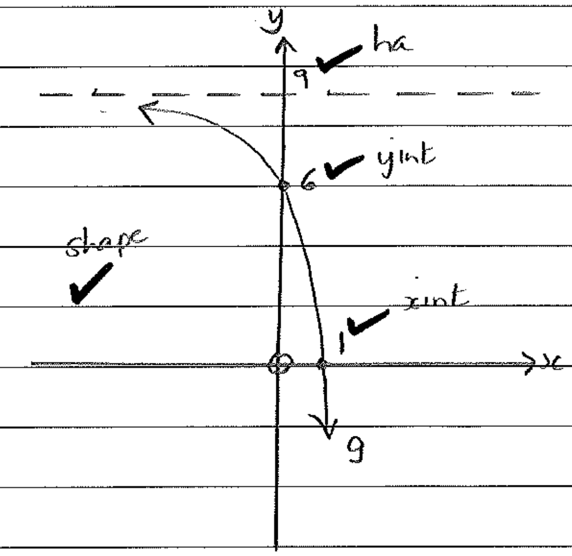
$9 = 3^{x+1}$

$3^2 = 3^{x+1}$ method $x+1 = \frac{\log 9}{\log 3}$

$\therefore 2 = x+1$

$1 = x$

ha: $y = 9$



5

6.2. $f: \quad y = \dfrac{5 + ax}{x + b}$

6.2.1. va: $x + b = 0$

$x = -b \qquad x = 1$

$\therefore -b = 1$

$\therefore b = -1$

So, $y = \dfrac{5 + ax}{x - 1}$

Sub $A(2;1)$

$1 = \dfrac{5 + a(2)}{2 - 1}$ sub

$1 = 5 + 2a$

$-4 = 2a$

$-2 = a$

$\therefore y = \dfrac{5 - 2x}{x - 1}$ **3**

6.2.2. $f(x) = \dfrac{5 - 2x}{x - 1}$

$$\begin{array}{r} -2 \\ x - 1 \,\big)\, -2x + 5 \\ \mp 2x \pm 2 \\ \hline +3 \end{array}$$

2

$\therefore f(x) = -2 + \dfrac{3}{x - 1}$

$= \dfrac{3}{x - 1} - 2$

OR

method

$f(x) = \dfrac{3}{x - 1} - 2$

$= \dfrac{3 - 2(x - 1)}{x - 1}$

$= \dfrac{3 - 2x + 2}{x - 1}$

$= \dfrac{5 - 2x}{x - 1}$

6.2.3. ha: $y = -2$

$\therefore B(2;-5)$ **2**

6.2.4.1. $f(x) = \dfrac{3}{x - 1} - 2$

$\therefore y = -(x - 1) - 2$

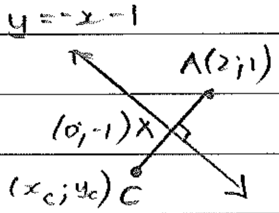
$y = -x + 1 - 2$

$y = -x - 1$

m c

2

6.2.4.2 $C(-2;-3)$ NO CA! **2**



Ac: $y = x + c$

Sub $A(2;1)$

$1 = 2 + c$

$-1 = c$

$\therefore y = x - 1$

X: $x - 1 = -x - 1$

$2x = 0$

$x = 0$

$\therefore y = 0 - 1$

$= -1$

$\therefore X(0;-1)$

$\dfrac{x_c + 2}{2} = 0$	$\dfrac{y_c + 1}{2} = -1$
$x_c + 2 = 0$	$y_c + 1 = -2$
$x_c = -2$	$y_c = -3$

7.1. $A\left(-\frac{5}{4};\ \frac{49}{8}\right)$ $\quad B(0;3)$

$\quad\ \ p \quad\ \ q$

$y = a(x-p)^2 + q$

$y = a\left(x-\left(-\frac{5}{4}\right)^2\right) + \left(\frac{49}{8}\right)$ ✓

$y = a\left(x+\frac{5}{4}\right)^2 + \frac{49}{8}$

sub $B(0;3)$

$3 = a\left(0+\frac{5}{4}\right)^2 + \frac{49}{8}$ ✓

$-\frac{25}{8} = a \cdot \frac{25}{16}$

$\dfrac{-\frac{25}{8}}{\frac{25}{16}} = a$

$-2 = a$ ✓

$\therefore y = -2\left(x+\frac{5}{4}\right)^2 + \frac{49}{8}$

$= -2\left(x+\frac{5}{4}\right)\left(x+\frac{5}{4}\right) + \frac{49}{8}$

$= -2\left(x^2 + \frac{5}{4}x + \frac{5}{4}x + \frac{25}{16}\right) + \frac{49}{8}$

$= -2\left(x^2 + \frac{5}{2}x + \frac{25}{16}\right) + \frac{49}{8}$ ✓

$= -2x^2 - 5x - \frac{25}{8} + \frac{49}{8}$

$= -2x^2 - 5x + 3$ **4**

7.2. $2x^2 + 5x - 3 = 2k$

$0 = -2x^2 - 5x + 2k + 3$

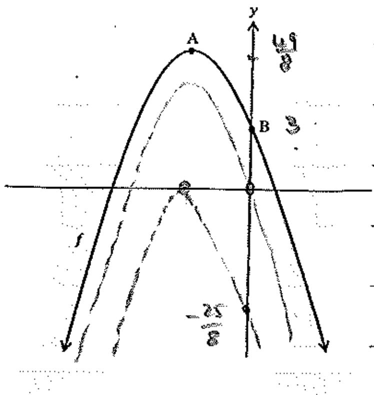
$y = -2x^2 - 5x + \underbrace{2k+3}_{c} \quad \therefore \uparrow\downarrow$

$-\frac{25}{8} < c < 0$

✓ $-\frac{25}{8} < 2k+3 < 0$ ✓ ✓

$-\frac{49}{8} < 2k < -3$

$-\frac{49}{16} < k < -\frac{3}{2}$ ✓ **4**



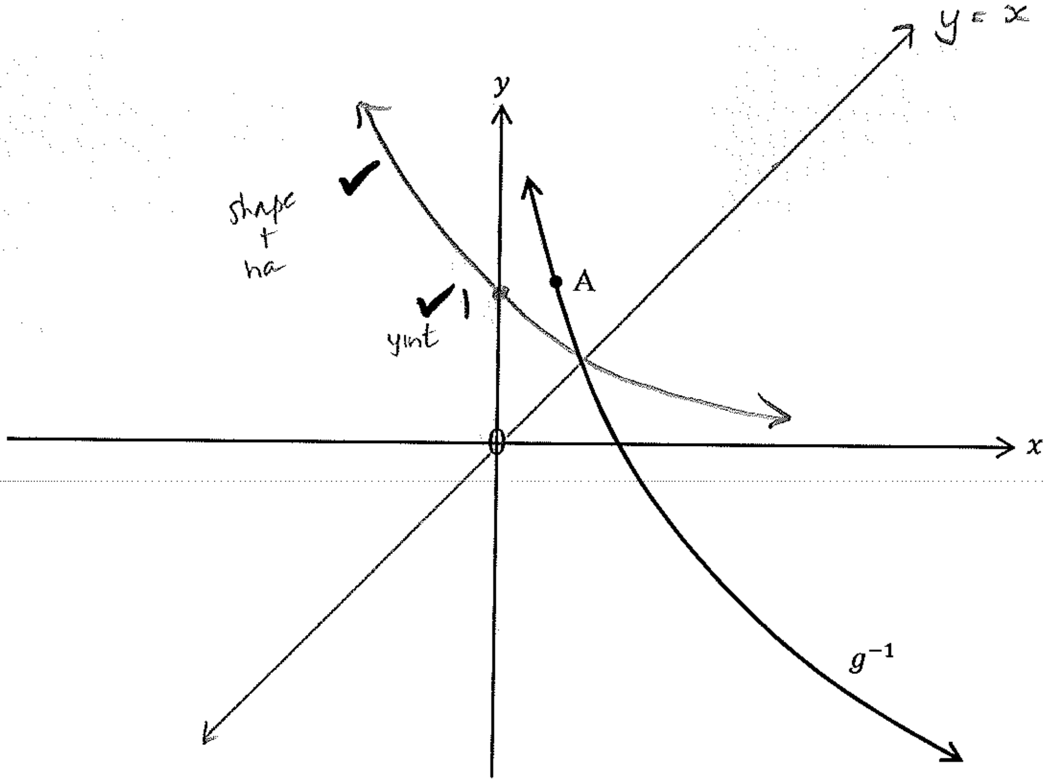
7.3.1. No ✓ **1**

7.3.2.1. f is a many to one function. ✓ **1**
(horizontal line cuts f more than once)

7.3.2.2. For f^{-1}, some x values have more than 1 y-value associated with them. ✓ **1**
(a place can be found where a vertical line crosses f^{-1} more than once)

ANSWER SHEET FOR QUESTION 8

8.



8.1 $g^{-1}: y = -\log_a x$

Sub $A(0,5;0,63)$

$0,63 = -\log_a 0,5$

$-0,63 = \log_a 0,5$

$a^{-0,63} = 0,5$

$(a^{-0,63})^{-\frac{1}{0,63}} = (0,5)^{-\frac{1}{0,63}}$

$a = 3$

4

8.2 $-\log_a x \geq 0,63$

$y_{g^{-1}} \geq 0,63$

$\therefore x \in (0;0,5]$

2

8.3 See graph

2

8.4 $h(x) = g^{-1}(x-2)$

$h = g^{-1}$ $2 \rightarrow$

$D_{g^{-1}}: x \in (0;\infty)$

$\therefore D_h: x \in (2;\infty)$

2

9.1.

$1 + i_{ea} = \left(1 + \frac{i_{nom}}{k}\right)^k$

$1 + i_{ea} = \left(1 + \frac{9}{400}\right)^4$

$i_{ea} = 0,093...$

$\therefore I_{ea} = 9,31\%$ pa

2

9.2.

$A = P(1 - i)^n$

$\frac{1}{3}x = x(1 - i)^8$

$\div x \quad (x \neq 0)$

$\frac{1}{3} = (1 - i)^8$

$\sqrt[8]{\frac{1}{3}} = 1 - i$

$i = 0,12...$

$I = 12,83\%$ pa

4

9.3.

$F = \frac{x[(1+i)^n - 1]}{i}$

fs i n

$11\,626 = \frac{300\left[\left(1 + \frac{5}{1200}\right)^n - 1\right]}{\frac{5}{1200}}$

$1,16... = \left(\frac{241}{240}\right)^n$

$n = \frac{\log 1,16...}{\log\left(\frac{241}{240}\right)}$

$= 36$ payments months

$\therefore 12k = 36$

$k = 3$

6

9.4. 1.

$1\,500\,000 - \frac{20}{100} \cdot 1\,500\,000$

$= 1\,500\,000 - 300\,000$

$= R\,1\,200\,000$

1

9.4. 2.

$P = \frac{x[1 - (1+i)^{-n}]}{i}$

fs i n

$1\,200\,000 = \frac{x\left[1 - \left(1 + \frac{8}{1200}\right)^{-240}\right]}{\frac{8}{1200}}$

$x = R\,10\,037,28$

3

9.4. 3.

$P = \frac{x[1 - (1+i)^{-n}]}{i}$

fs n

$= \frac{10\,037,28\left[1 - \left(1 + \frac{8}{1200}\right)^{-60}\right]}{\frac{8}{1200}}$

$= R\,495\,022,92$

3

OR

$A = P(1+i)^n$

$= 1\,200\,000\left(1 + \frac{8}{1200}\right)^{180}$

$= 3\,968\,305,77...$

$F = \frac{x[(1+i)^n - 1]}{i}$

$= \frac{10\,037,28\left[\left(1 + \frac{8}{1200}\right)^{180} - 1\right]}{\frac{8}{1200}}$

$= 3\,473\,282,52...$

$\therefore OB = A - F$

$= R\,495\,023,25$

10.1. $f(x) = -x^2 + 3$

$f(x+h) = -(x+h)^2 + 3$

$= -(x^2 + 2xh + h^2) + 3$

$= -x^2 - 2xh - h^2 + 3$

$f'(x) \longleftarrow$ if not stated -1

$= \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{-x^2 - 2xh - h^2 + 3 - (-x^2 + 3)}{h}$ ✓

$= \lim_{h \to 0} \dfrac{-x^2 - 2xh - h^2 + 3 + x^2 - 3}{h}$

$= \lim_{h \to 0} \dfrac{-2xh - h^2}{h}$ ✓

$= \lim_{h \to 0} \dfrac{h(-2x - h)}{h}$ ✓

$= \lim_{h \to 0} (-2x - h)$ ✓

$= -2x - (0)$

$= -2x$ ✓ **5**

10.2. i : $y = -\frac{3}{x} + 1$

$x = -1 \therefore y = -\frac{3}{-1} + 1$

$= 4$

$\therefore (-1; 4)$ ✓

$x = 2 \therefore y = -\frac{3}{2} + 1$

$= -\frac{1}{2}$

$\therefore (2; -\frac{1}{2})$ ✓

av grad $= \dfrac{\Delta y}{\Delta x}$

$= \dfrac{-\frac{1}{2} - 4}{2 - (-1)}$

$= -\frac{3}{2}$ ✓ **3**

10.3. $x + 2 = 0 \therefore x = -2$

$R = g(-2)$ ✓

$3 = 3(-2)^4 + a(-2) - 5$ ✓

$2a = 40$

$a = 20$ ✓ **3**

10.4.1. $3x - 2 = 0 \therefore x = \frac{2}{3}$

$h(\frac{2}{3}) = 12(\frac{2}{3})^3 - 17(\frac{2}{3})^2 + 36(\frac{2}{3}) - 20$ ✓

$= 0$ ✓

$\therefore 3x - 2$ is a factor **2**

10.4.2. $12x^3 - 17x^2 + 36x - 20$

$= (3x - 2)(4x^2 - 3x + 10)$

$-8x^2$

$\therefore -9x^2$

$= -17x^2$

$\therefore$ other factor is

$4x^2 - 3x + 10$

✓ ✓ **2**

tier 1 prob | tier 2 prob | outcomes

11.1. 1. $P(A \cap B) = 0{,}45$ ✓

$\neq 0$

$\therefore$ No ✓, A and B are NOT mutually exclusive. **2**

11.1. 2. $P(A) \times P(B)$

$= (0{,}3 + 0{,}45)(0{,}45 + 0{,}15)$

$= \left(\frac{3}{4}\right)\left(\frac{3}{5}\right)$ ✓

$= \frac{9}{20}$

$= 0{,}45$

$= P(A \text{ and } B)$

$\therefore$ Yes ✓, A and B are independent. **2**

11.2. At least two sports

$= \frac{2+3+4}{2} + \frac{5}{3}$

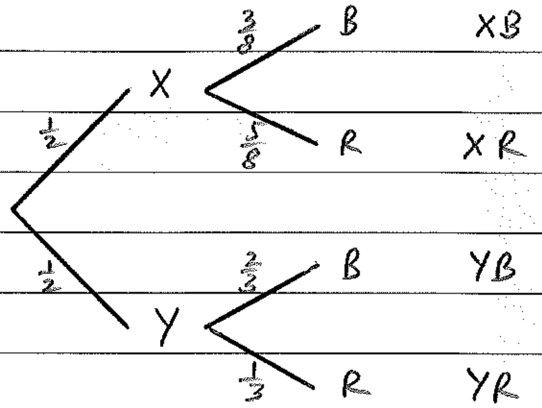
$= 14$ ✓ **1**

Note :

11.1. 1/2 Yes No, but no valid reason 0/2

11.2. $\frac{14}{45}$ X 0/1

11.3. 1. ✓ ✓ ✓



3

11.3. 2. $P(B)$

$= P(XB \text{ or } YB)$

$= P(XB) + P(YB)$

$= \frac{1}{2} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{2}{3}$

$= \frac{3}{16}$ ✓ $+ \frac{1}{3}$ ✓

$= \frac{25}{48}$ ✓ 0,52 **3**